# Byzantine Robust Optimization: A dual approach

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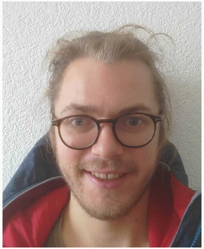

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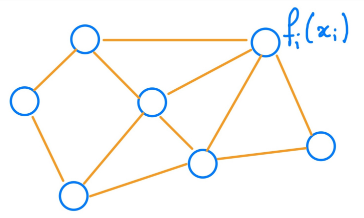

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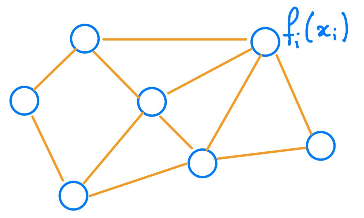
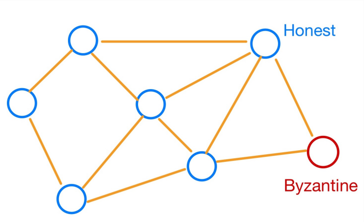


# Decentralized Optimization under Byzantine corruption

- $n$ computing units in a network with
  - Local cost functions;
  - Local memory.
- Units cooperate to find a global solution:

$$x^* \in \mathrm{argmin}_{x\in\mathbb{R}^d} \left\{ f_h(x) := \sum_{i \text{ honest}} f_i(x) \right\}.$$

- Some unknown units are *Byzantine*, i.e malicious and omniscient.



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Solved using Gradient Descent.

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2. Share it with neighbors,
3. **Average** messages received,
4. Actualize your parameter using it.

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$\Rightarrow$ lever for limiting influence of Byzantines units

Questions ?

# Appendix

# Gossip dual algorithm

*For each units $i$:*
*Initialize $y_i^0 = 0$*

1. compute $x_i^t := \nabla f_i^*(y_i^t)$,
2. share $x_i^t$ with his neighbors,
3. actualization of $y_i^t$:
$$y_i^{t+1} := y_i^t - \eta \sum_{j \sim i} \left( x_i^t - x_j^t \right) .$$

$\implies$ **Duality gives practical decentralized algorithms !**